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Development of Generalized Aeroservoelastic Reduced Order Models

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Design of modern control laws motivates the creation of state-space models from aeroservoelastic models. Conventional methods for generating state-space models either result in a large number of states, or reduce models into inconsistent forms. In this development, a method is created that generates consistent state-space models. Specifically, the terms of the state-space matrices have a similar basis across a range of flight conditions. This enables meaningful interpolation of the state-space models away from the analyzed data points. Models that are generated in this method and applied across a range of flight conditions are referred to as generalized reduced-order models. With this method, reduced-order models (ROMs) can be generated that characterize a vehicle throughout its operation, and control laws can be simulated across a range of flight conditions.

I. Introduction

The pursuit of increasingly advanced vehicles creates a demand for a strong capability for generating vehicle math models. This demand is well exhibited through the example of high speed transport aircraft design. Significant study conducted during the NASA High Speed Research (HSR) program identified the importance of performing active control for flutter suppression, gust load alleviation, and ride quality enhancement[1,2]. These requirements apply across the transonic flight regime, making the problem significantly more challenging[3,4,5]. Ongoing research is aimed at developing and testing active control laws for this class of vehicle[6]. To enable design of these control laws, state space representations of the vehicle's aeroservoelastic behavior must be created. Methods for generating reduced order state space models for aeroservoelastic control law design are therefore in high demand.

Existing methods for generating state space models can be categorized into test based methods and analysis based methods. Test based methods are capable of calculating state space models directly from transfer function data collected in test[7]. While the direct use of test data offers high accuracy, this data is not available during the vehicle design process and cannot be easily recomputed for changes in configuration. Methods based on analysis data have been developed for varying levels of analysis fidelity. Systems using potential flow have generally been based on the Doublet-Lattice Method[8,9] as seen in numerous analytical tools and applications. This approach is can be extended to use corrected unsteady aerodynamic data[10,11,12] to generate models that are linearized about high fidelity data points. More recently, generation of highly accurate ROMs has been performed directly from Euler or Navier-Stokes CFD data[13,14,15]. Conversion of ASE data to state space models can be accomplished by approximating the unsteady aerodynamic content using Rational Function Approximation (RFA)[16] or by matching the aeroelastic behavior directly using the P-Transform technique[17,18,19]. The RFA approach has the drawback of generating a high-order model, and model reduction is often employed to create a lower order ROM. Neither the P-Transform approach nor the reduced models from the RFA approach retain the physical consistency of the model states. Specifically, the meaning of a given term in the state space matricies is not consistent for calculations at various flight conditions. As such, interpolation of the state space model among flight conditions is not valid.

Using a conventional approach such as Roger's RFA initially generates a high order state space model. In order to convert this model into a suitable form for control law synthesis and evaluation, the model order must be reduced. A powerful method for performing this reduction is conversion of the state space model into a balanced realization followed by truncation. A balanced realization is a means of ordering the modes of a model based on the modes that are the most observable and the most controllable[20,21,22]. This ordering is accomplished by the calculation of the

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controllability and observability Gramians. Once the modes are ordered in this way order reduction is accomplished by truncating the modes which are the least observable and controllable.

Development of the Generalized Reduced Order Model generation (GROM) system fills a need for an approach that can be queried across multiple flight conditions. The current implementation of this method is based on Doublet-Lattice unsteady aerodynamic approach to which correction factors can be applied. Conversion of aerodynamic data to state space form is accomplished though the use of Roger's Rational Function Approximation. The significant divergence from previous approaches lies in the reduction of the high-order state space model. In this approach, the controllability and observability Gramians are calculated across a range of flight conditions. These quantities are then summed resulting in an indication of the significance of a given mode to a range of flight conditions. Using these summations, the model is consistently converted into a realization that is balanced in a generalized sense. Due to the fact that the same transformation is applied to the state space models for each flight condition, the models maintain consistency. This consistency has been enforced through the structure inherent in assembling the full order state space model and preserved through the use of a common transformation matrix. It is in this form that the insignificant states are truncated leaving a family of reduced order models that can be interpolated or curve fit.

II. Generalized Reduced Order Model Generation Process

The process for generating generalized reduced order models consists of: converting the structural model to modal coordinates, obtaining a high order state space representation of the aerodynamic model, calculating Gramians at multiple flight conditions, and reducing model order in a physically consistent manner. This process is shown in the flowchart below.

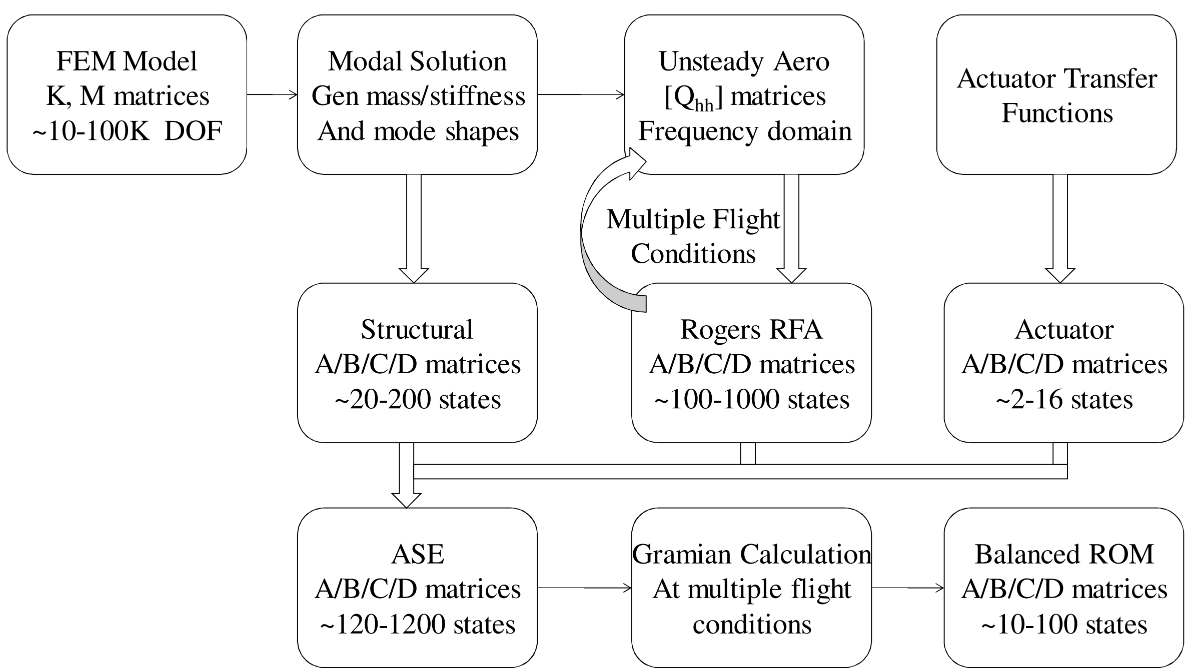

FIGURE 1: FLOW CHART FOR GENERALIZED REDUCED ORDER MODEL GENERATION

A. Modal Coordinate Reduction and State Space Representation of Structural Dynamics

Obtaining the state space model representation of the structural dynamics is accomplished in a conventional manner. The input to this procedure is a finite element model capable of generating mass (M), stiffness (K) and possibly damping (C) matrices. Using this data the modal solution solves the generalized eigenvalue problem:

$$K\Phi = \omega^2 M\Phi$$

Using this data, the model is transformed to modal coordinates:

$$\tilde{K} = \Phi^T K \Phi, \quad \tilde{C} = \Phi^T C \Phi, \text{ and } \tilde{M} = \Phi^T M \Phi$$

The order of this representation is determined by the number of eigenvalues used. From this point, the model is converted to state space format.

$$\begin{Bmatrix} \dot{\xi} \\ \ddot{\xi} \end{Bmatrix} = \begin{bmatrix} 0 & I \\ -\tilde{M}^{-1}\tilde{K} & -\tilde{M}^{-1}\tilde{C} \end{bmatrix} \begin{Bmatrix} \xi \\ \dot{\xi} \end{Bmatrix} + \begin{bmatrix} 0 \\ \tilde{M}^{-1} \end{bmatrix} F_{aero} + \begin{bmatrix} 0 \\ \tilde{M}^{-1} \end{bmatrix} \begin{Bmatrix} 0 \\ F_{actuator} \end{Bmatrix}$$

where ξ is the generalized displacement. This state space model has the generalized motion as output and the aerodynamic and actuator forces as inputs. This form is suitable for combining with an aerodynamic model.

B. High Order State Space Representation of Aerodynamics

Roger's Rational Function Approximation (RFA) method[16] is used to obtain a state space representation of the unsteady aerodynamics. This is obtained in the classical manner by solving a curve fit to the generalized aerodynamic influence coefficients (AICs). This is expressed as:

$$Q_{mn}(p) = A_{mn}^0 + pA_{mn}^1 + p^2 A_{mn}^2 + \sum_{l=1}^{nlags} \frac{pB_{mn}^l}{p + \beta^l}$$

where the A_i and B_i terms are determined based on a least squares solution. This transfer function matrix is then converted to state space form.

$$\begin{aligned} \dot{z} &= A_A z + B_A u \\ F_{aero} &= C_A z + D_A u \end{aligned}$$

where z is a vector of aerodynamic lag states, u is a vector of structural deflections, velocities, and accelerations in the generalized coordinates, and F_{aero} is a vector of the resulting aerodynamic forces in the generalized degrees of freedom. The A, B, C, and D matrices are constructed from the parameters of the rational function approximation as follows:

$$A_A = \frac{2V}{c} diag(-\beta^1, -\beta^2, \ldots, -\beta^{nlag})$$

$$B_A = \begin{bmatrix} 0 & \begin{bmatrix} \hat{B}_A & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \hat{B}_A \end{bmatrix} & 0 \end{bmatrix}$$ where $\hat{B}_A$ is a nlag x 1 matrix populated with values of 1.0

$$C_A = \begin{bmatrix} B^1 & \ldots & B^{nlag} \end{bmatrix}$$

$$D_A = \begin{bmatrix} A^0 & \dfrac{c}{2V} A^1 & \left(\dfrac{c}{2V}\right)^2 A^2 \end{bmatrix}$$

C. Actuator Model

In addition to modeling the structure and aerodynamics, an actuator model is included in the aeroservoelastic state space model. This is done by modeling the actuator as a feedback control system with gain k and delay constant a. Choosing $k = M_{RR}\omega_{act}^2$ and $a = 2\zeta_{act}/\omega_{act}$ (where M_{RR} is the hinge inertia, ω_{act} and ζ_{act} are the natural frequency and damping of the actuator) gives a second order system with unity steady state gain. The actuator moment can be written as:

$$M_{act} = k(\delta_{cmd} - \delta - a\dot{\delta})$$

Or:

$$R^T F_{act} = R^T k\left(\delta_{cmd} - R\xi - aR\dot{\xi}\right)$$

Where R is a matrix converting the generalized deflection vector ξ into unit (radian) deflection of the control surfaces, eg:

$$\delta = R\xi$$

This actuator model is suitable for inclusion in the dynamic equation for the system.

D. Assembly of High Order Aeroservoelastic State Space Model

Once the aerodynamic and structural state space models have been constructed, the aerodynamic model can then be connected to the structural model as feedback. The generalized AICs are input to the structural model and output from the aerodynamic model, the modal deflections are input to the aerodynamic model and output from the structural model. This results in the following state space model:

$$\begin{Bmatrix} \dot{\xi} \\ \ddot{\xi} \\ \dot{z} \end{Bmatrix} = \begin{bmatrix} 0 & I & 0 \\ -\hat{M}^{-1}\hat{K} & -\hat{M}^{-1}\hat{C} & \hat{M}^{-1}C_A \\ 0 & B_A^1 & A_A \end{bmatrix} \begin{Bmatrix} \xi \\ \dot{\xi} \\ z \end{Bmatrix} + \begin{bmatrix} 0 \\ \hat{M}^{-1}R^T k \\ 0 \end{bmatrix} \{\delta_{cmd}\} .$$

$$y = \begin{bmatrix} \phi & 0 & 0 \end{bmatrix} \begin{Bmatrix} \xi \\ \dot{\xi} \\ z \end{Bmatrix} .$$

$$\ddot{y} = \begin{bmatrix} -\phi\hat{M}^{-1}\hat{K} & -\phi\hat{M}^{-1}\hat{C} & \phi\hat{M}^{-1}C_A \end{bmatrix} \begin{Bmatrix} \xi \\ \dot{\xi} \\ z \end{Bmatrix} + \begin{bmatrix} \phi\hat{M}^{-1}R^T k \end{bmatrix} \{\delta_{cmd}\} .$$

where y is the vector of nodal deflections in physical coordinates, R is a matrix converting the generalized deflection vector ξ into unit deflection of the control surfaces, and:

$$\begin{aligned} \hat{M} &= \tilde{M} - D_A^2 \\ \hat{C} &= \tilde{C} - D_A^1 + R^T kaR \\ \hat{K} &= \tilde{K} - D_A^0 + R^T kR \end{aligned}$$

E. Model Reduction for a Range of Flight Conditions

The process detailed above results in high-order state-space matrices (A,B,C,D) that are each valid for a particular flight condition. A transformation matrix must be identified that will operate on the state-space models for each flight condition. If flight conditions are parameterized in terms of Mach number (M), dynamic pressure (q), and mass ratio (μ), this transformation can be written as:

$$A'(M,q,\mu) = T^{-1} A(M,q,\mu)\, T$$

where the transformation matrix, T, transforms the A matrix so that the resulting states are ordered in terms of their significance for control law design.

The quantities considered in this model reduction are the sums of the controllability and observability Gramians across the prescribed range of flight conditions. These are expressed as:

$$W_c'(0,t_f) = \sum_{i=1,n} \int_0^{t_i} e^{\tau A_i} B B^T e^{\tau A_i^T} d\tau \quad \text{and} \quad W_o'(0,t_f) = \sum_{i=1,n} \int_0^{t_i} e^{\tau A_i^T} C^T C e^{\tau A_i} d\tau$$

where Gramians are summed over n flight conditions. Note that these summations provide an indication of the controllability and observability across flight conditions as long as the Gramians for each flight condition are positive semi-definite.

As is the case for the conventional balanced realization, the transformation matrix of interest can be determined by solving a generalized eigenvalue problem. For the case of this generalized balancing, this problem can be written as:

$$W'_O T = \sigma^2 W'^{-1}_C T$$

Once this transformation has been solved, the generalized balanced realization of the state-space matrices are determined as:

$$A_i' = T^{-1} A_i T$$
$$B_i' = T^{-1} B_i$$
$$C_i' = C_i T$$
$$D_i' = D_i$$

Since the state matrices have been transformed by the same matrix, when they are truncated to the same number of states, they will be in a form suitable for interpolation.

III. Demonstration on Aeroservoelastic Example Problem

A. Problem Description

The example problem selected for this development is based on the AGARD 445.6[23,24]. The configuration has a taper ratio of 0.7 and an aspect ratio of 3.5. A control surface has been added at the trailing edge. The AGARD 445.6 wing is a classical test case in computational aeroelasticity, and its aeroelastic behavior is well understood. By adding a control surface, it is possible to generate a simple, yet well understood ASE example model for GROM testing and V&V. This configuration is shown in the Figure 2.

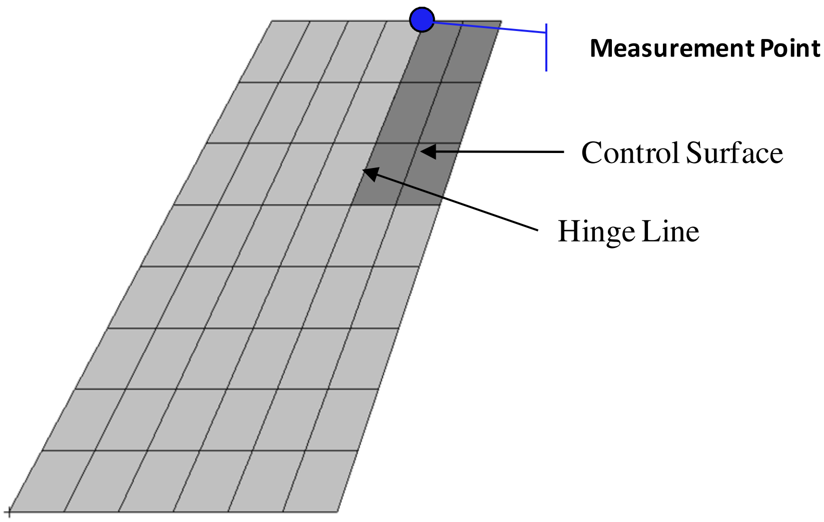

FIGURE 2: GROM EXAMPLE PROBLEM BASED ON AGARD 445.6

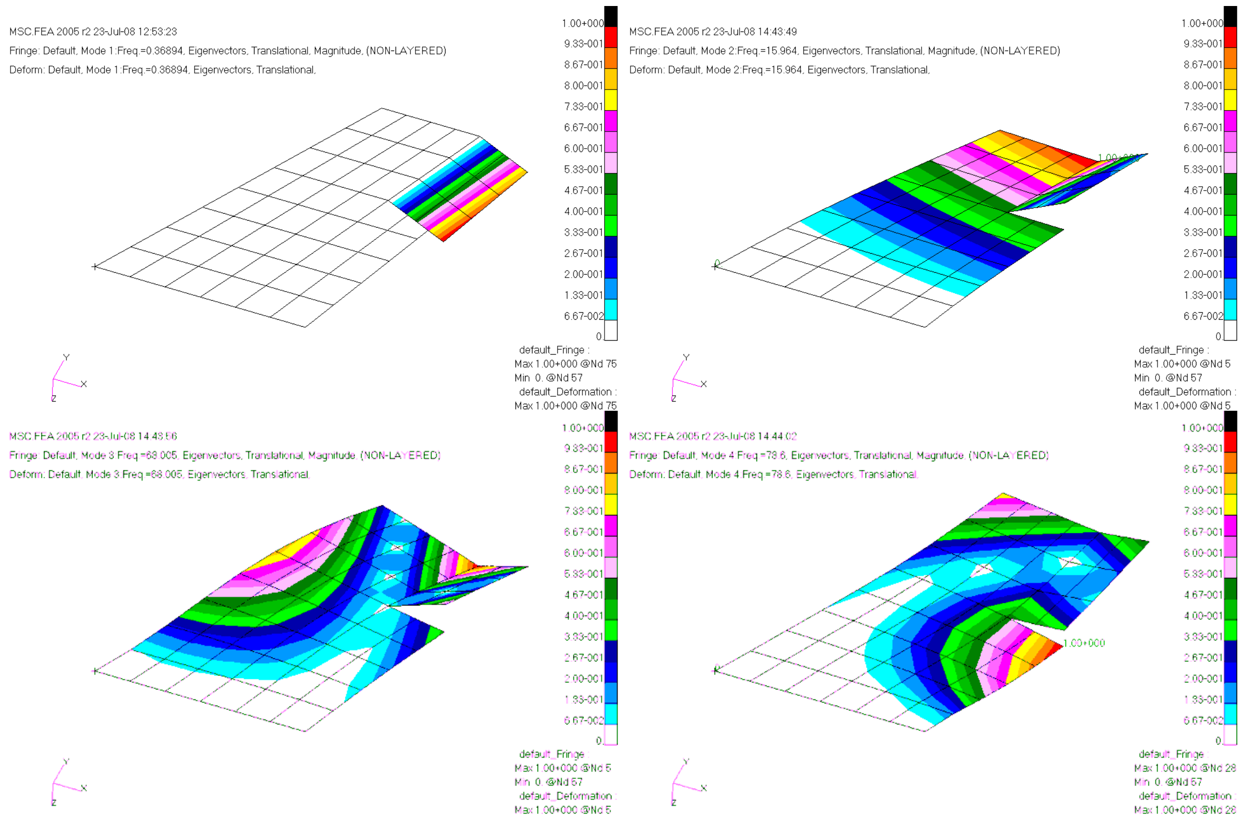

FIGURE 3: CONTROL SURFACE MODE AND FIRST THREE ELASTIC MODES

B. Results

In order to demonstrate the full capability of GROM, a ROM was generated for a M=0.5 condition based on Nastran analyses at M=0.3 and M=0.7. All conditions were run at a dynamic pressure of 200psi. The ROM at M=0.5 was generated through the GROM process from the state-space models for the M=0.3 and M=0.7 conditions reduced using a common transformation. The accuracy of this approach was assessed based on comparisons of the transfer function from control surface excitations to deflections at the wing tip measured at the hinge line location. The frequency response functions for both the interpolated model and the full order Nastran models were calculated in the frequency domain. Figure 4 shows a comparison of the M=0.5 GROM result to a full Nastran analysis performed a posteriori for validation purposes. The Nastran results for M=0.3 and M=0.7 are included in the figure to give an indication the magnitude of the interpolation. Note that the interpolated ROM matches the full Nastran solution reasonably well throughout the frequency range spanned by the first seven elastic modes. It is noted, however, that the magnitudes of some of the pole and root regions show noticeable differences. This motivates the use of higher order interpolation methods during the future development.

A particularly significant behavior seen in Figure 4 is the ability of the interpolated state space model (GROM model) to match the character of Nastran frequency response function even where it differs from the two Nastran solutions used as the basis for interpolation. This behavior is evident near 40 Hz where the results at M=0.5 differ significantly from the results at M=0.3 and M=0.7. This behavior is seen in both the GROM result and the full order Nastran result. It is clear that this type of result could not be obtained simply by interpolating the frequency response data, and as such it demonstrates some of the potential utility of the GROM method.

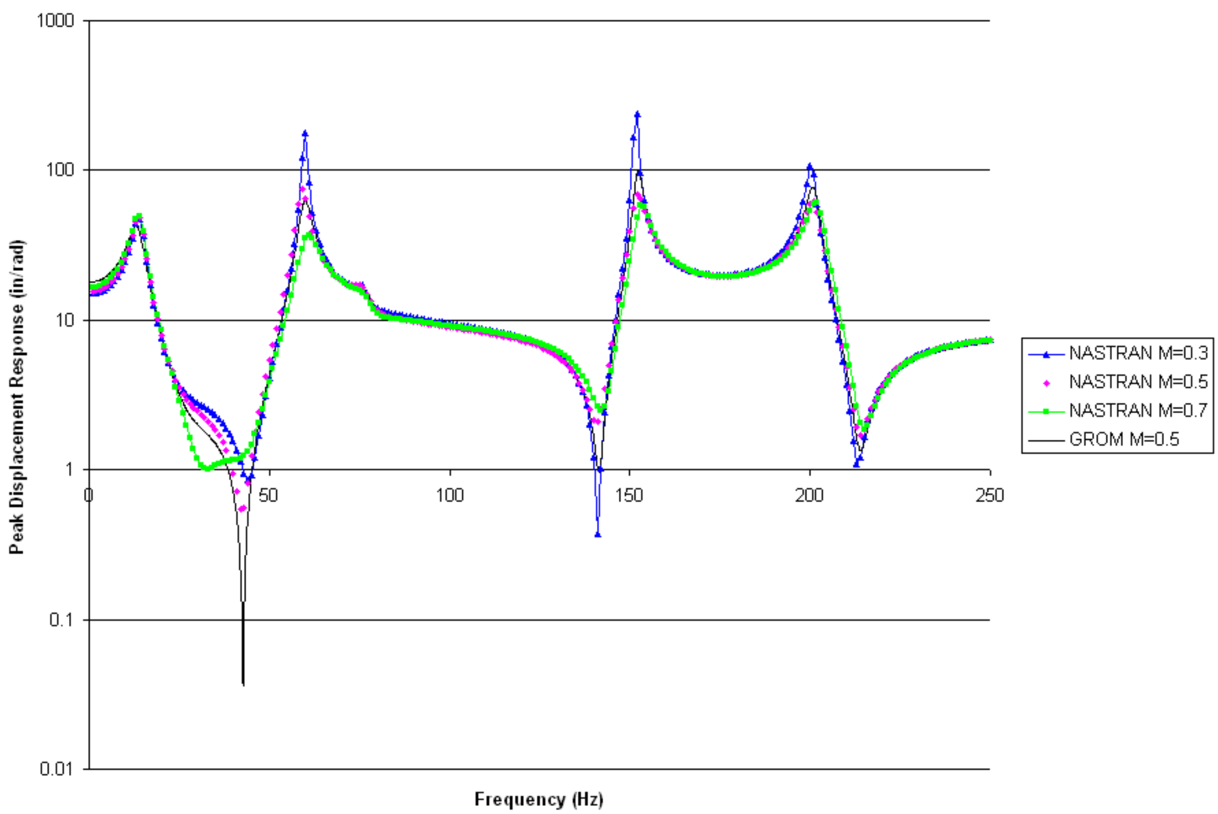

FIGURE 4: COMPARISON OF GROM RESULT TO FULL NASTRAN RESULT FOR AGARD 445.6 BASED EXAMPLE PROBLEM

IV. Conclusion

A method has been developed for generating aeroservoelastic reduced order models that have consistent structure across flight conditions. These models can be interpolated across flight conditions and are referred to as generalized reduced order models. The accuracy of this approach has been assessed through comparison of aeroservoelastic transfer functions from an interpolated model and a full order model. These results demonstrated good correlation and methods for improving the accuracy of interpolated models were identified. Potential topics for future work in this area include the use of higher order response surface models for representing the terms of the state space models, application to aeroservoelastic models representing compressibility effects, and application in the transonic regime.

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