52nd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference
 19th
4 - 7 April 2011, Denver, Colorado

AIAA 2011-2064

Efficient Simulation of Structural Dynamic Systems with Discrete Nonlinearities

Kevin M. Roughen*, Jinhua Huang†, Daniel C. Hammerand‡, Daniel S. Stuewe§
M4 Engineering, Inc., Long Beach, CA, 90807

Shawn R. Hertz**
Naval Air Warfare Center Weapons Division, China Lake, CA, 93555

Dynamic loading environments often involve complex loading and boundary conditions. For the case of externally mounted aircraft stores, qualification testing involves simulating this environment with relatively simple test fixtures and load application shakers. Simulation using Finite Element Analysis (FEA) is used to ensure that the test conditions are representative of the service environment. In this development, a method for efficiently simulating test and service conditions is presented. The component mode synthesis method[1,2] is used to significantly reduce the computational requirements of solving nonlinear store vibration problems while retaining a straightforward model structure at user specified boundary locations. While the component mode synthesis method provides an efficient coordinate transformation, the physical meaning of the degrees of freedom at the boundary are preserved. Nonlinear stiffness functions at the boundary degrees of freedom are added after the linear potion of the model has been reduced. The resulting set of nonlinear equations is integrated in time. Accuracy and computational expense are evaluated through comparison to full-order nonlinear solutions from existing commercial software.

I. Introduction

An important aspect of store qualification testing is tuning of laboratory tests to accurately represent the service use environment. This is done through simulation of both the test and service environments. These simulations result in quantification of the number of times critical values of load or stress are exceeded. This cyclic loading data can be combined with fatigue strength data for a given material to predict whether the structure can withstand the prescribed load spectrum. Note that while model tuning is performed by comparing test and analysis results for the same conditions, laboratory test tuning is performed on analytical results from the test and service environments (Figure 1).

Comparison of the simulated cyclic loading data between test and service conditions provides valuable insight to engineers. This insight includes indication of whether the test is more or less severe than the service environment at various structural details. When combined with trade studies on the test configuration, this data enables tuning of laboratory tests to more accurately represent service loads. Nonlinear effects may have a significant effect on the accuracy of these findings. The interface of stores with the carrying aircraft is a well known source of nonlinearity. These interfaces are designed for effective release of the store and generally result in nonlinear stiffness at the interface. This development presents a new method for efficiently including that nonlinear stiffness.

---

* Vice President of Engineering, Member, AIAA.
† Senior Engineer, Member, AIAA.
‡ Senior Engineer, Member, AIAA.
§ Senior Engineer, Senior Member, AIAA.
** Supervisory Mechanical Engineer, Member, AIAA.

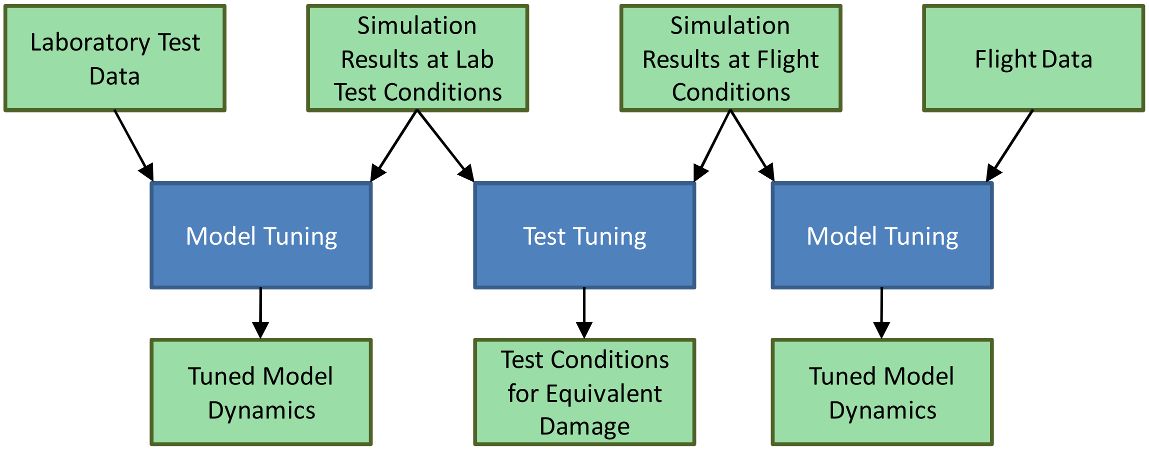

Figure 1. Use of nonlinear simulation results in model tuning and test tuning.

The use of Finite Element Models (FEM) to represent structures is a mature and powerful technology. Nonlinear elements have been used in dynamic simulations for a range of problems including nonlinearities associated with structural joints[3,4]. While this inclusion of nonlinear elements into dynamic solutions is certainly possible, the complexity and computational runtime associated with this approach has often limited it to research applications. The development presented here combines time integration of nonlinear equations with component mode synthesis model reduction techniques[1,2] to significantly reduce the computational expense associated with the inclusion of structural nonlinearities.

II. Technical Approach

A. Finite Element Model Reduction (Component Mode Synthesis Method)

While linear dynamic analysis of models with tens or even hundreds of thousands of degrees of freedom can be routinely solved, inclusion of nonlinear components in such models will result in unacceptable computational runtimes and hardware requirements. This challenge is addressed through the use of component mode synthesis[1,2] to significantly reduce the number of degrees of freedom present during solution of the nonlinear equations of motion.

Component mode synthesis accomplishes a reduction in the number of degrees of freedom present when solving the problem through an efficient change of coordinates. This is done starting from the classic dynamic finite element problem in physical coordinates:

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = \{F\} \tag{1}$$

The problem is partitioned into degrees of freedom corresponding to the boundary, and the left over structure. These coordinate sets are denoted as the R-set (removed degrees of freedom corresponding to the boundary) and the L-set (left over degrees of freedom)[5]. The transformed system is written as:

$$\begin{bmatrix} M_{RR} & M_{RL} \\ M_{LR} & M_{LL} \end{bmatrix} \begin{Bmatrix} \ddot{U}_R \\ \ddot{U}_L \end{Bmatrix} + \begin{bmatrix} C_{RR} & C_{RL} \\ C_{LR} & C_{LL} \end{bmatrix} \begin{Bmatrix} \dot{U}_R \\ \dot{U}_L \end{Bmatrix} + \begin{bmatrix} K_{RR} & K_{RL} \\ K_{LR} & K_{LL} \end{bmatrix} \begin{Bmatrix} U_R \\ U_L \end{Bmatrix} = \begin{Bmatrix} F_R \\ F_L \end{Bmatrix} \tag{2}$$

From this point, the component mode synthesis approach develops a matrix of boundary node functions (*B*) and fixed based mode shapes (*Φ*). These matrices are calculated based on the full mass and stiffness matrices for user specified boundary degrees of freedom. The resulting system equations of motion are:

$$\begin{bmatrix} B^T M B & B^T M \Phi \\ \Phi^T M B & I \end{bmatrix} \begin{Bmatrix} \ddot{U}_R \\ \ddot{q}_m \end{Bmatrix} + \begin{bmatrix} B^T C B & 0 \\ 0 & \Phi^T C \Phi \end{bmatrix} \begin{Bmatrix} \dot{U}_R \\ \dot{q}_m \end{Bmatrix} + \begin{bmatrix} B^T K B & 0 \\ 0 & \omega_0^2 \end{bmatrix} \begin{Bmatrix} U_R \\ q_m \end{Bmatrix} = [B \quad \Phi]^T \begin{Bmatrix} F_R \\ F_L \end{Bmatrix} \tag{3}$$

In this equation, U_R represents displacements at the boundary and q_m represents generalized (modal) deflections. The solution is conducted in these coordinates and then transformed to physical coordinates for extraction of physical quantities including displacement and stress using:

$$\begin{Bmatrix} U_R \\ U_L \end{Bmatrix} = \begin{bmatrix} B & \Phi \end{bmatrix} \begin{Bmatrix} U_R \\ q_m \end{Bmatrix} \tag{4}$$

Note that equations of motion for the component mode synthesis approach (Eq. (3)) have the advantage of containing only degrees of freedom required to define the boundary and the modal behavior of the structure. As such, a significant reduction in the degrees of freedom (DOF) of the problem is possible. For example, an engineer could reduce a 10,000 DOF store model to 45 DOF provided that the store was well represented by the first 30 modes and the boundaries of interest consisted of two 6 DOF hanger interfaces and three 1 DOF shaker interfaces. This approach preserves the dynamic behavior of the structure and contains physical coordinates at the boundaries for interfacing with force application or nonlinear elements.

B. Modeling of Nonlinear Components

After choosing appropriate boundary degrees of freedom, reduced order equations of motion are developed which contain physical coordinates at the interfaces with nonlinear components (Eq. (3)). Once the stiffness is defined as a nonlinear function of the physical boundary coordinates, it is added to the equations of motion as follows:

$$\begin{bmatrix} B^T MB & B^T M\Phi \\ \Phi^T MB & I \end{bmatrix} \begin{Bmatrix} \ddot{U}_R \\ \ddot{q}_m \end{Bmatrix} + \begin{bmatrix} B^T CB & 0 \\ 0 & \Phi^T C\Phi \end{bmatrix} \begin{Bmatrix} \dot{U}_R \\ \dot{q}_m \end{Bmatrix} + \begin{bmatrix} B^T KB & 0 \\ 0 & \omega_0^2 \end{bmatrix} \begin{Bmatrix} U_R \\ q_m \end{Bmatrix} = \begin{bmatrix} B & \Phi \end{bmatrix}^T \begin{Bmatrix} F_R \\ F_L \end{Bmatrix} + \begin{bmatrix} B & \Phi \end{bmatrix}^T \begin{Bmatrix} f(U_R) \\ 0 \end{Bmatrix} \tag{5}$$

where $f(U_R)$ is a vector of functions describing the force as a function of displacement (nonlinear stiffness) on the boundary degrees of freedom.

Nonlinear damping could also be accommodated, resulting in the following equations of motion:

$$\begin{bmatrix} B^T MB & B^T M\Phi \\ \Phi^T MB & I \end{bmatrix} \begin{Bmatrix} \ddot{U}_R \\ \ddot{q}_m \end{Bmatrix} + \begin{bmatrix} B^T CB & 0 \\ 0 & \Phi^T C\Phi \end{bmatrix} \begin{Bmatrix} \dot{U}_R \\ \dot{q}_m \end{Bmatrix} + \begin{bmatrix} B^T KB & 0 \\ 0 & \omega_0^2 \end{bmatrix} \begin{Bmatrix} U_R \\ q_m \end{Bmatrix} = \begin{bmatrix} B & \Phi \end{bmatrix}^T \begin{Bmatrix} F_R \\ F_L \end{Bmatrix} + \begin{bmatrix} B & \Phi \end{bmatrix}^T \begin{Bmatrix} f(U_R, \dot{U}_R) \\ 0 \end{Bmatrix} \tag{6}$$

where $f(U_R, \dot{U}_R)$ is a function of boundary velocities as well as boundary displacements.

It is important to emphasize that the nonlinear functions are described in terms of physical position and velocity at the interfaces. This means that functions developed through tests of the nonlinear components can be included in a logical manner. Note that this preservation of physically relevant boundary coordinates has been performed while significantly reducing the number of degrees of freedom, and thus nonlinear equations of motion.

C. Solution of Nonlinear Dynamic System

Various software modules exist for solution of a coupled series first order ordinary differential equations. These equations have the form:

$$\{\dot{y}\} = \{f(\{y\}, t)\} \tag{7}$$

where the first time derivative of a vector function ($\{\dot{y}\}$) is expressed as a function of the vector state ($\{y\}$) as well as time (t). All other quantities are constant during the solution.

The nonlinear equations of motion are transformed into a series of first order differential equations. Starting from Eq. (6), simplify the notation by defining the C-set as the combination of the R and m sets and introducing M_C, C_C, K_C, and T:

$$[M_C]\{\ddot{U}_C\} + [C_C]\{\dot{U}_C\} + [K_C]\{U_C\} = [T]\{F_C(t)\} + [T]\{f(U_C, \dot{U}_C)\} \tag{8}$$

The mass matrix is a square matrix of low order. For physically realistic problems, it is generally well conditioned for inversion. Inversion of the mass matrix and pre-multiplication results is:

$$\{\ddot{U}_C\} + [M_C]^{-1}[C_C]\{\dot{U}_C\} + [M_C]^{-1}[K_C]\{U_C\} = [M_C]^{-1}[T]\{F_C(t)\} + [M_C]^{-1}[T]\{f(U_C, \dot{U}_C)\} \tag{9}$$

Defining a function $g() = M_C^{-1} T f()$, and simplifying:

$$\{\ddot{U}_C\} = -[M_C^{-1}C_C]\{\dot{U}_C\} - [M_C^{-1}K_C]\{U_C\} + [M_C^{-1}T]\{F_C(t)\} + \{g(U_C, \dot{U}_C)\} \tag{10}$$

This can be rewritten in first order form as:

$$\begin{Bmatrix} \dot{U}_C \\ \ddot{U}_C \end{Bmatrix} = \begin{bmatrix} 0 & I \\ -M_C^{-1}K_C & -M_C^{-1}C_C \end{bmatrix} \begin{Bmatrix} U_C \\ \dot{U}_C \end{Bmatrix} + [M_C^{-1}T]\{F_C(t)\} + \{g(U_C, \dot{U}_C)\} \tag{11}$$

Note that Eq. (11) is in the form of Eq. (7). The state vector (y) is composed of the displacements (U_C) and velocities ($\dot{U}_C$) in the coordinates developed during component mode synthesis. Mass, stiffness, damping, and force depend only on constant parameters as well as the state ($[U_C\ \dot{U}_C]^T$) and time (t).

From this form, the equations are integrated forward in time from initial displacement and velocity conditions with prescribed force-time histories and nonlinear force-displacement profiles. This is implemented using the ordinary differential equation integration function from the SciPy Python library (ODEINT). This function operates directly on equations of the form of Eq. (7). Function ODEINT uses the Livermore Solver for Ordinary Differential Equations (LSODE) created at Lawrence Livermore National Laboratory[6,7,8]. LSODE automatically detects whether an ordinary differential equation (ODE) system is stiff or non-stiff at each time step. If it is non-stiff, the multi-step Adams predictor/corrector method is applied, whereas if it is stiff, backward differentiation formula (BDF) methods are used.

D. Validation of Nonlinear Dynamic Solver

Two examples have been employed to demonstrate the accuracy of the nonlinear solver. The first example is for a system of second order differential equations given below:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \{\ddot{U}_C\} + \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \{U_C\} = \begin{Bmatrix} 1.5e^t \\ 0 \\ 2t^2 + 2t + 6 \end{Bmatrix}$$
$$U_C = \begin{Bmatrix} 1 \\ 0 \\ 2 \end{Bmatrix}, \quad \dot{U}_C = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} \tag{12}$$

The analytical and ODEINT solutions for Eq. (12) are shown in Figure 2. The ODEINT solution is nearly identical to the analytical solution for this first example problem.

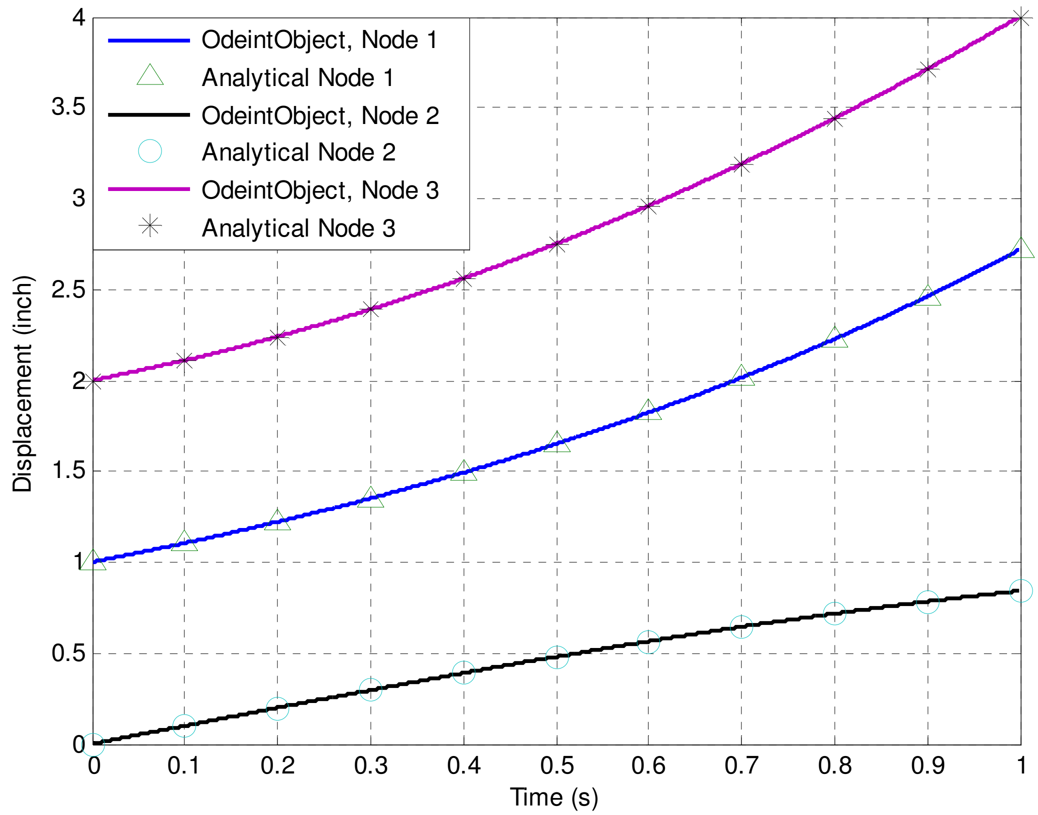

Figure 2 Analytical and ODEINT solutions for differential equations (Eq. (12)).

III. Demonstration

A. Problem Description

The second example problem is based on a 2-D representation of a notional store vibration test with nonlinear joint conditions. The store is a beam with mass and stiffness uniformly distributed. The stiffness and mass properties are as follows:

- Length: 185.24 inches
- Mass: 1514 lb
- Modulus: 30 Msi
- Section moment of inertia: 161.6 in^4
- Interface with aircraft at the forward and aft points (Figure 3)

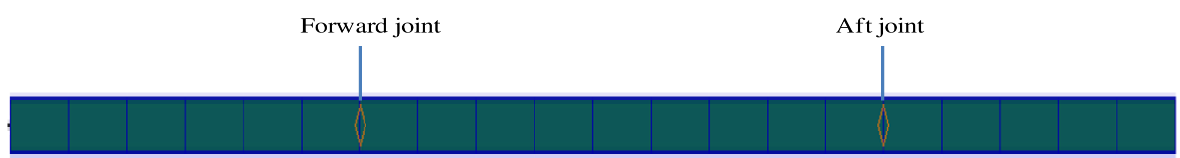

Figure 3 FEM model of notional store.

All antisymmetic degrees of freedom are constrained. The aircraft interfaces at the forward and aft joints have a free-play nonlinearity in the vertical and for-aft translational degrees of freedom. The gap distance is $5x10^{-5}$ in. The modal frequencies for the case with free interfaces are shown in Table 1.

Table 1 Modal frequency results with free interface.

Mode number	Modal Frequency (Hz.)
1	0.0
2	0.0
3	0.0
4	248.3
5	1866.8
6	6110.1
7	7101.2
8	19206.3

B. Results

A vertical sine dwell shaker load is applied to the forward joint point. This load has an amplitude of 6,400 lb and a frequency of 3 Hz. Using the process described above, the model size is first reduced using the Craig-Bampton method and then solved using ODEINT. This simulation is performed for a duration of 10 seconds. The results for the forward node vertical displacement are compared to a full order nonlinear simulation performed in MSC.Nastran showing excellent correlation (Figure 4).

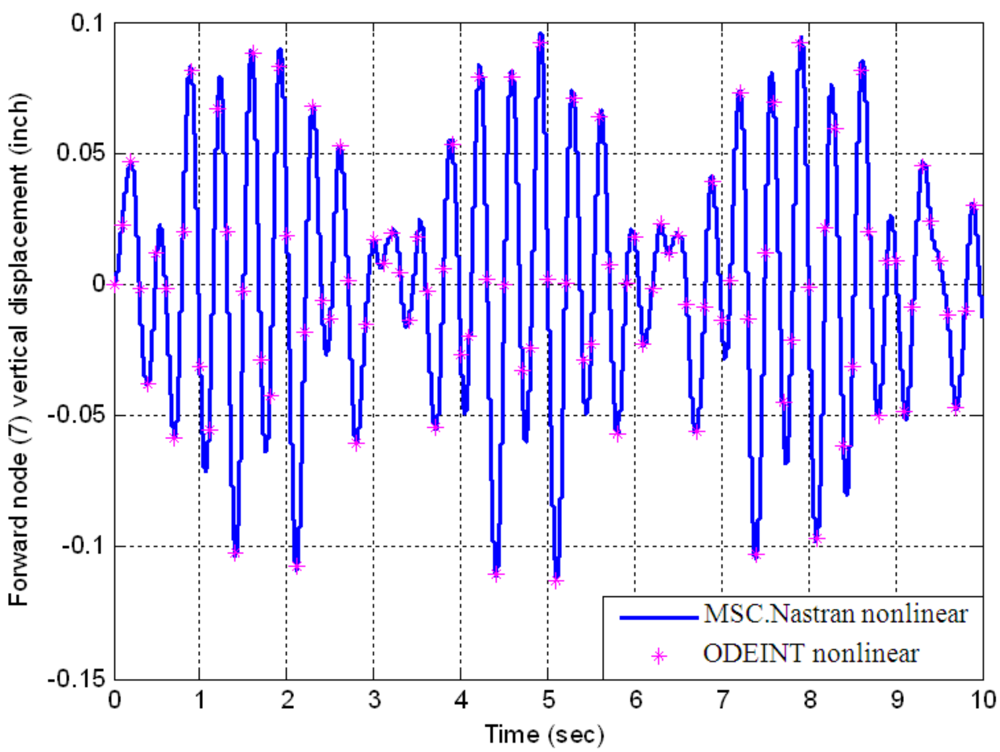

Figure 4: The forward node vertical displacement from ODEINT and MSC.Nastran solutions for the second example with nonlinear free-play joints.

IV. Conclusion

A method has been implemented for efficiently simulating structural dynamic systems with discrete nonlinearities. Simulation of a beam example with nonlinear supports showed excellent correlation with a full order simulation. The use of modal representations of the linear portions of the structure offers the potential for substantial reductions in runtime and memory requirements relative to conventional nonlinear simulation.

V. Acknowledgement

Funding for this development has been provided by the Naval Air Weapons Center at China Lake. The technical monitor is Shawn Hertz.

VI. References

1. Hurty, W. C. "Dynamic Analysis of Structural Systems Using Component Modes," AIAA Journal, Vol. 3, No. 4, April, 1965, pp. 678-685.
2. Craig, R. R., and Bampton, M. C. C. "Coupling of Substructures for Dynamic Analysis," AIAA Journal, Vol. 6, No. 7, July, 1968.
3. Lyle, K. H., Jackson, K. E., and Fasanella E. L. "Simulation of Aircraft Landing Gears with a Nonlinear Dynamic Finite Element Code," Journal of Aircraft, Vol. 39, No. 1, 2002.
4. Gaul, L. and Lenz, J. "Nonlinear Dynamics of Structures Assembled by Bolted Joints," Acta Mechanica, Vol. 125, No. 1, 1997.
5. MSC.Software Corporation, *Basic Dynamic Analysis User's Guide,* Santa Ana, 2004.
6. Hindmarsh, Alan C, Serial Fortran Solvers for ODE Initial Value Problems, [Online] Lawrence Livermore National Laboratory. https://computation.llnl.gov/casc/odepack/odepack_home.html,
7. Hindmarsh, A. C. "ODEPACK, A Systematized Collection of ODE Solvers," *Scientific Computing, Vol. 1 of IMACS Transactions on Scientific Computation,* 1983, pp. 55-64.
8. Petzold, L. R. "Automatic Selection of Methods for Solving Stiff and Nonstiff Systems of Ordinary Differential Equations," *SIAM Journal on Scientific and Statistical Computing,* 1983, pp. 136-148.