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# Development of an Efficient Uncertainty Quantification Framework Applied to an Integrated Spacecraft System

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**The objective of the study described in this paper was to develop an efficient uncertainty quantification framework capable of analyzing uncertainty in integrated spacecraft system models. Specifically, this paper discusses the capabilities of the developed framework and the results when applied to the multidisciplinary analysis of a reusable launch vehicle (RLV). This particular framework is capable of efficiently propagating mixed (inherent and epistemic) uncertainties through complex simulation codes. The Second-Order Probability Theory utilizing a stochastic response surface obtained with Point-Collocation Non-Intrusive Polynomial Chaos was used for the propagation of the mixed uncertainties. This particular methodology was applied to the RLV analysis, and the uncertainty in the output parameters of interested was obtained in terms of intervals at various probability levels. This study has also demonstrated the feasibility of the developed uncertainty quantification framework for efficient propagation of mixed uncertainties in the analysis of complex aerospace systems.**

## I. Introduction

Uncertainties are generally ubiquitous in the analysis and design of highly complex engineering systems. Uncertainties can arise from the lack of knowledge in physical modeling (epistemic uncertainty), inherent variations in the systems (aleatory uncertainty), and numerical errors in the computational procedures used for analysis. It is important to account for all of these uncertainties in applications such as robust and reliable design of multi-disciplinary aerospace systems. A reusable launch vehicle (RLV) is a highly complex aerospace system, which represents a cost viable option for access to space missions due to its reusability aspect. Since an RLV system is composed of various subsystems which must work together to serve an over-arching purpose or mission objective/constraint, a multidisciplinary approach for the analysis and design of RLV systems is required.

Uncertainties are generally present in the models used in each discipline of an RLV multidisciplinary analysis framework. It is important to account for all of these uncertainties for accurate and reliable estimates of the RLV system performance. The primary purpose of this paper will be to demonstrate an *efficient approach* for uncertainty quantification in the multidisciplinary analysis of a RLV, which has a mixture of aleatory (inherent) and epistemic input uncertainties. Uncertainty quantification will be performed on various output variables of interest for an RLV system such as the weight and vehicle cross-range. For the propagation of mixed (aleatory-epistemic) uncertainty, Second-Order Probability Theory utilizing Point-Collocation Non-Intrusive Polynomial Chaos (NIPC)[1] will be used.[2-3] In general, the NIPC methods, which are based on the spectral representation of uncertainty, are computationally more efficient than traditional Monte Carlo methods for moderate number of uncertain variables and can give highly

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accurate estimates of various uncertainty metrics. In addition, they treat the deterministic model (e.g, an RLV system) as a black box and the uncertainty information in the output is approximated with a polynomial expansion, which is constructed using a number of deterministic solutions, each corresponding to a sample point in random space. Therefore, the NIPC methods become a perfect candidate for the uncertainty quantification in the numerical solutions, which are computationally expensive and complex.

Previously, M4 Engineering developed the Multidisciplinary Optimization Object Library (MOOL) as part of a phase II SBIR effort funded by NASA Glenn Research Center. An object-oriented Multidisciplinary Analysis Optimization (MDAO) framework is an automated analysis, optimization, and virtual test system that allows (1) consideration of interactions between disciplines during analysis, (2) automated execution of multiple analysis codes on different computers, (3) incorporation of test data to improve accuracy of analysis models, and (4) optimization of vehicle parameters to achieve superior performance. In the MOOL project, M4 developed a suite of common MDAO objects that can be used in multiple framework environments to handle common tasks encountered in integration of multidisciplinary analysis and optimization problems. Specifically, as an example application, a high-alpha RLV system was integrated and analyzed. In the current study, the same MOOL high-alpha RLV system will be implemented for uncertainty quantification (UQ) analysis.

Currently, M4 Engineering and Missouri S&T have partnered to develop an UQ framework. This framework is built upon an Uncertainty Module previously developed by M4 Engineering. One of the key objectives of this project is to implement the Non-Intrusive Polynomial Chaos (NIPC) expansion methods within the framework. This framework was developed in Python language[13] and primarily serves as an outer analysis layer. The utility of this framework is through the efficient application of NIPC methods to any user-specified system, which can be executed from a command line.

In the following section, a brief description of the various types of uncertainties found in complex numerical simulations is given. In Section III, a brief overview on the theory behind Point-Collocation Non-Intrusive Polynomial Chaos will be given. Next in Section IV, an efficient approach to propagate mixed aleatory and epistemic uncertainties through a simulation code using NIPC and Second-Order Probability will be outlined. In Section V, the details of the uncertainty quantification framework will be given. In Section VI, the MOOL RLV system will be described in detail along with preliminary uncertainty quantification results. Finally in Section VII, the conclusions and the main objectives of our future work are given.

## II. Types of Uncertainties in Computational Simulations

As described in Oberkampf et. al.,[4] there can be three different types of uncertainty and error in a computational simulation: (1) aleatory uncertainty, (2) epistemic uncertainty, and (3) numerical error. The term aleatory uncertainty describes the inherent variation of a physical system. Such variation is due to the random nature of input data and can be mathematically represented by a probability density function if substantial experimental data are available for estimating the distribution (uniform, normal, etc.). The variation of the free-stream velocity or manufacturing tolerances can be given as examples for aleatory uncertainty in a stochastic external aerodynamics problem. The aleatory uncertainty is sometimes referred as irreducible uncertainty due to its nature.

Epistemic uncertainty in a non-deterministic system originates due to ignorance, lack of knowledge, or incomplete information (such as the values of transport quantities in high temperature hypersonic flow simulations). The key feature of this definition is that the fundamental cause is incomplete information of some characteristics of the system. As a result, an increase in knowledge or information can lead to a decrease in the epistemic uncertainty. Therefore, epistemic uncertainty is referred to as reducible uncertainty. As shown by Oberkampf and Helton,[5] modeling of epistemic uncertainties with probabilistic approaches may lead to inaccurate predictions in the amount of uncertainty in the responses due to the lack of information on the characterization of uncertainty as probabilistic. One approach to characterize the epistemic uncertain variables is to use intervals. The upper and lower bounds on the uncertain variable can be prescribed using either limited experimental data or expert judgment.

Numerical error is defined as a recognizable deficiency in any phase or activity of modeling and simulation that is not due to the lack of knowledge. If errors cannot be well-characterized, then they must be treated as part of the epistemic uncertainties. The discretization error in spatial or temporal domain originating from the numerical solution of partial differential equations that describes a physical model in a discretized computational space (mesh) can be given as an example of numerical uncertainty.

## III. The Point-Collocation NIPC

The polynomial chaos is a stochastic method, which is based on the spectral representation of the uncertainty. An important aspect of spectral representation of uncertainty is that one may decompose a random function (or variable) into separable deterministic and stochastic components. For example, for any random variable (i.e., $\alpha^*$) such as velocity, density or pressure in a stochastic fluid dynamics problem, we can write,

$$\alpha^*(\vec{x},\vec{\xi}) = \sum_{i=0}^{P} \alpha_i(\vec{x})\,\Psi_i(\vec{\xi}), \qquad \textbf{(1)}$$

where $\alpha_i(\vec{x})$ is the deterministic component and $\Psi_i(\vec{\xi})$ is the random basis function corresponding to the $i^{th}$ mode. Here we assume $\alpha^*$ to be a function of deterministic independent variable vector $\vec{x}$ and the n-dimensional random variable vector $\vec{\xi} = (\xi_1,...,\xi_n)$, which has a specific probability distribution. The discrete sum is taken over the number of output modes,

$$P + 1 = \frac{(n+p)!}{n!\,p!}, \qquad \textbf{(2)}$$

which is a function of the order of polynomial chaos ($p$) and the number of random dimensions ($n$). The basis function ideally takes the form of multi-dimensional Hermite Polynomial to span the n-dimensional random space when the input uncertainty is Gaussian (unbounded), which was first used by Wiener[8] in his original work of polynomial chaos. Legendre (Jacobi) and Laguerre polynomials are optimal basis functions for bounded (uniform) and semi-bounded (exponential) input uncertainty distributions respectively in terms of the convergence of the statistics. Different basis functions can be used with different input uncertainty distributions (See Xiu and Karniadakis[9] for a detailed description), however the convergence may be affected depending on the basis function used. The detailed information on polynomial chaos expansions can be found in Walters and Huyse[10] and Hosder et. al.[1]

To model the uncertainty propagation in computational simulations via polynomial chaos with the intrusive approach, all dependent variables and random parameters in the governing equations are replaced with their polynomial chaos expansions. Taking the inner product of the equations, (or projecting each equation onto $i^{th}$ basis) yield $P + 1$ times the number of deterministic equations which can be solved by the same numerical methods applied to the original deterministic system. Although straightforward in theory, an intrusive formulation for complex problems can be relatively difficult, expensive, and time consuming to implement. To overcome such inconveniences associated with the intrusive approach, non-intrusive polynomial chaos formulations have been considered for uncertainty propagation.

The collocation based NIPC method starts with replacing the uncertain variables of interest with their polynomial expansions given by Equation 1. Then, *P+1* vectors ($\vec{\xi}_i = \{\xi_1, \xi_2, ..., \xi_n\}_k, \; k = 0,1,2,...,P$) are chosen in random space for a given PC expansion with *P +1* modes and the deterministic code is evaluated at these points. With the left hand side of Equation 1 known from the solutions of deterministic evaluations at the chosen random points, a linear system of equations can be obtained:

$$\begin{bmatrix} \Psi_0(\vec{\xi}_0) & \Psi_1(\vec{\xi}_0) & \cdots & \Psi_P(\vec{\xi}_0) \\ \Psi_0(\vec{\xi}_1) & \Psi_1(\vec{\xi}_1) & \cdots & \Psi_P(\vec{\xi}_1) \\ \vdots & \vdots & \ddots & \vdots \\ \Psi_0(\vec{\xi}_P) & \Psi_1(\vec{\xi}_P) & \cdots & \Psi_P(\vec{\xi}_P) \end{bmatrix} \begin{Bmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_P \end{Bmatrix} = \begin{Bmatrix} \alpha^*(\vec{\xi}_0) \\ \alpha^*(\vec{\xi}_1) \\ \vdots \\ \alpha^*(\vec{\xi}_P) \end{Bmatrix} \tag{3}$$

The spectral modes ($\alpha_k$) of the random variable $\alpha^*$ are obtained by solving the linear system of equations given above. Using these, mean ($\mu_{\alpha^*}$) and the variance ($\sigma^2_{\alpha^*}$) of the solution can be obtained by

$$\begin{aligned} \mu_{\alpha^*} &= \alpha_0 \\ \sigma^2_{\alpha^*} &= \sum_{i=1}^{P} \alpha_i^2 \left\langle \Psi_i^2(\vec{\xi}) \right\rangle \end{aligned} \tag{4}$$

The solution of the linear problem given by Equation 3 requires *P +1* deterministic function evaluations. If more than *P +1* samples are chosen, then the over-determined system of equations can be solved using the Least Squares method. Hosder et. al.[11] investigated this option by increasing the number of collocation points in a systematic way through the introduction of a parameter $n_p$ defined as

$$n_p = \frac{number\ of\ \ samples}{P+1}. \tag{5}$$

In the solution of stochastic model problems with multiple uncertain variables, they have used $n_p = 1, 2, 3,$ and *4* to study the effect of the number of collocation points (samples) on the accuracy of the polynomial chaos expansions. Their results showed that using a number of collocation points that is twice more than the minimum number required ($n_p=2$) gives a better approximation to the statistics at each polynomial degree. This improvement can be related to the increase of the accuracy of the polynomial coefficients due to the use of more information (collocation points) in their calculation. The results of the stochastic model problems also indicated that for problems with multiple random variables, improving the accuracy of polynomial chaos coefficients in NIPC approaches may reduce the computational expense by achieving the same accuracy level with a lower order polynomial expansion.

## IV. Mixed Aleatory-Epistemic Uncertainty Propagation

In this study, Second-Order Probability[2-3] is utilized to propagate mixed (aleatory and epistemic) uncertainty through multidisciplinary analysis framework. Second-Order Probability uses an inner loop and an outer sampling loop as described in Figure 1. In the outer loop, a specific value for the epistemic variable is prescribed and then passed down to the inner loop. Any traditional aleatory uncertainty method may then be used to perform aleatory uncertainty analysis in the inner loop for the specified value of the epistemic uncertain variable. The Second-Order Probability will give interval bounds for the output variable of interest at different probability levels. Each iteration of the outer loop will produce a cumulative distribution function (CDF) based on the aleatory uncertainty analysis in the inner loop. Thus, if there are 100 samples in the outer loop, then 100 different CDF curves will be generated. One major advantage of Second-Order Probability is that it is easy to separate and identify the aleatory and epistemic uncertainties. On the other hand, the two sampling loops can make this method computationally expensive especially if traditional sampling techniques, such as Monte Carlo, are used for the uncertainty propagation.

The current study utilizes an efficient approach for the propagation of mixed uncertainties using the framework based on Second-Order Probability. With this approach, the stochastic response is represented with a polynomial chaos expansion on both epistemic and aleatoric variables. In this study, Point-Collocation NIPC is used to construct the stochastic response surface although other NIPC methods (i.e., quadrature or sampling based) can be also used. The optimal basis functions are used for the aleatoric variables whereas Legendre polynomials are used for the epistemic uncertain variables. It should be noted that the use of Legendre polynomials should not imply a uniform probability assignment to the epistemic variables. This choice is made due to the bounded nature of epistemic uncertain variables. Once the stochastic response surface is formed, at fixed values of epistemic uncertain variables, the stochastic response values can be evaluated for a large number of samples randomly produced based on the probability distributions of the aleatoric input uncertainties (inner loop of Second-Order Probability). This procedure will produce a single cumulative distribution function. By repeating the inner loop procedure for a large number of epistemic uncertain variables sampled from their corresponding intervals (outer loop of Second-Order Probability), a population of cumulative distribution functions can be obtained which can be used to calculate the bounds of the stochastic response at different probability levels. Due to the analytical nature (polynomial) of the stochastic response, the described procedure will be computationally efficient, especially compared to the approaches based on direct MC sampling which require a large number of deterministic simulations.

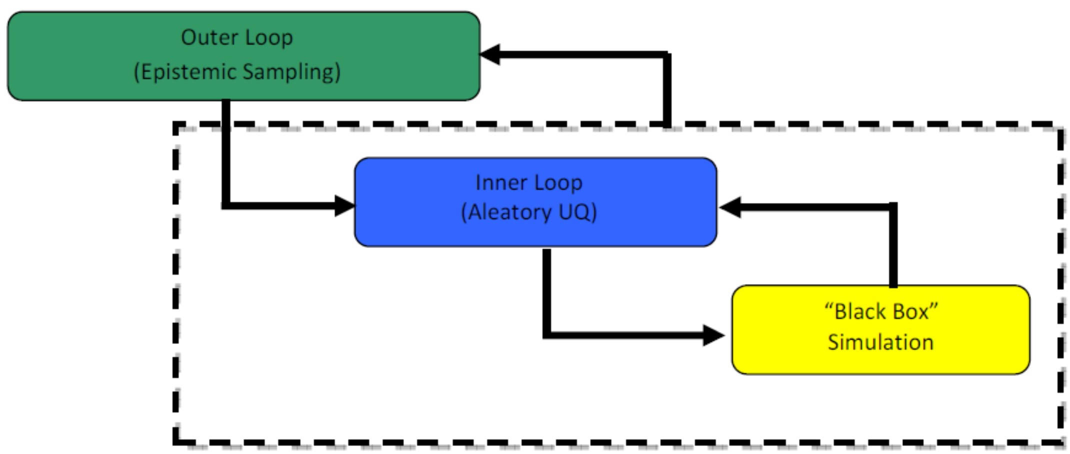


**Figure 1. Schematic of second-order probability.**

## V. Uncertainty Quantification Framework

### A. Overview

The UQ Framework was developed as a Python class within the M4.Uncertainty module. This allows the framework to have access to the various UQ techniques, algorithms, and visualization schemes existing within the M4.Uncertainty module. The framework was designed to serve as a generalized outer analysis layer for a given user-specified system (see Figure 2).

This approach allows the framework to analyze and interact with the inputs and outputs of any user-specified system that can be executed from a command line. The UQ Framework consists of three main methods:

1. preProcess():
    - Generates sample values for input random variables and writes them to the Framework OutputFolder.
2. systemExecute():
    - Executes the user-specified system by calling it from the command line.
3. postProcess():
    - Performs the UQ analysis on the system output data.

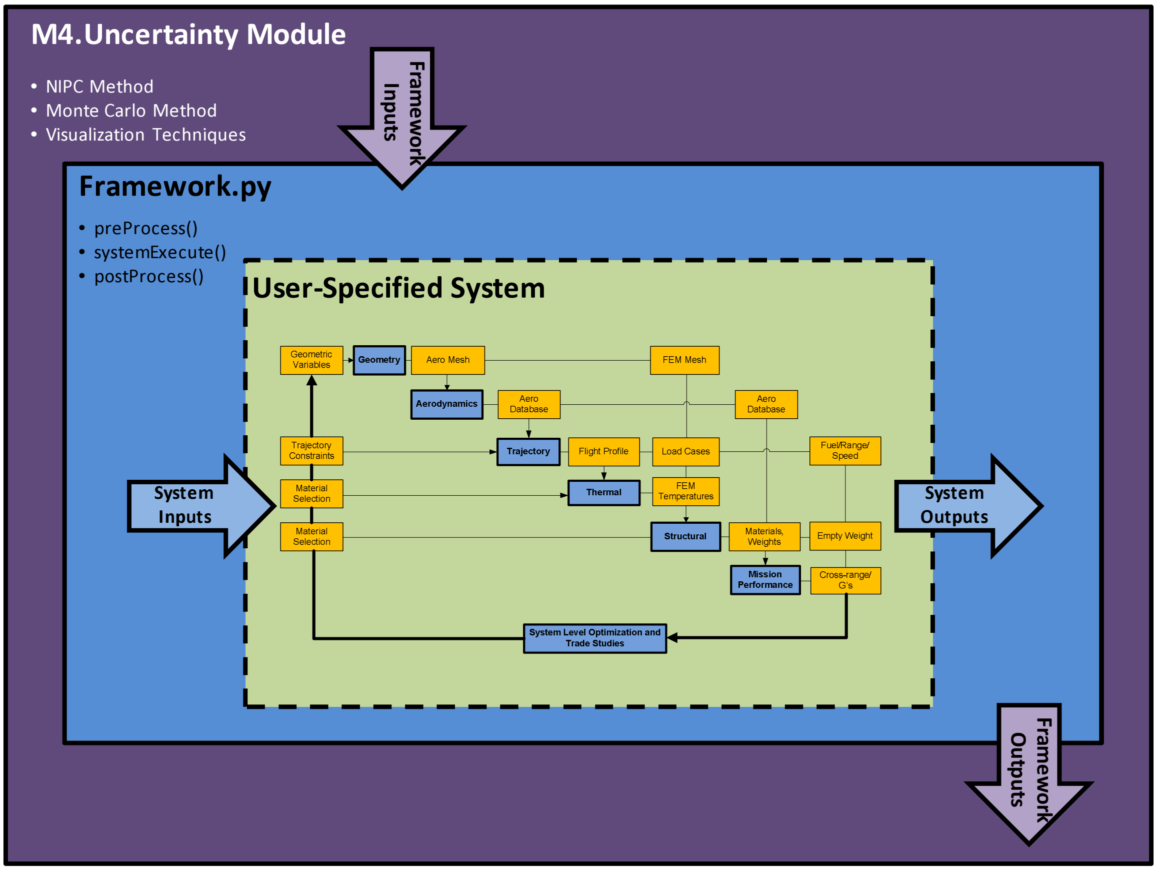


**Figure 2. Overview of UQ Framework**

## B. Framework Operation

The UQ Framework operation process can be summarized by the following:

- Begin preProcess()
    - Generate and write required number of sample values for input random variables to a file to be used by systemExecute()
    - Organize and write UQ method (i.e. NIPC) input data to a file to be used by postProcess()
- Generate output values for each sample
    - For each sample value, wrap the user-specified system inputs
    - Begin systemExecute()
    - Collect, organize, and write comma-separated, column-formatted output values from user-specified system outputs to a file

- Begin postProcess()
    - Invoke UQ post process method,
        - For NIPC method, solve matrix equation for alpha coefficients for each output to determine the mean
        - Compute Gauss-quadrature to determine variance and standard deviation for output variables of interest
        - Write cumulative distribution function (CDF) and probability density function (PDF) plot data to a file in UQ Framework output folder
- Generate CDF and PDF plots
    - Using previously generated plot data, create and save PNG files for each output variable of interest

Figure 3 shows a schematic of this process.

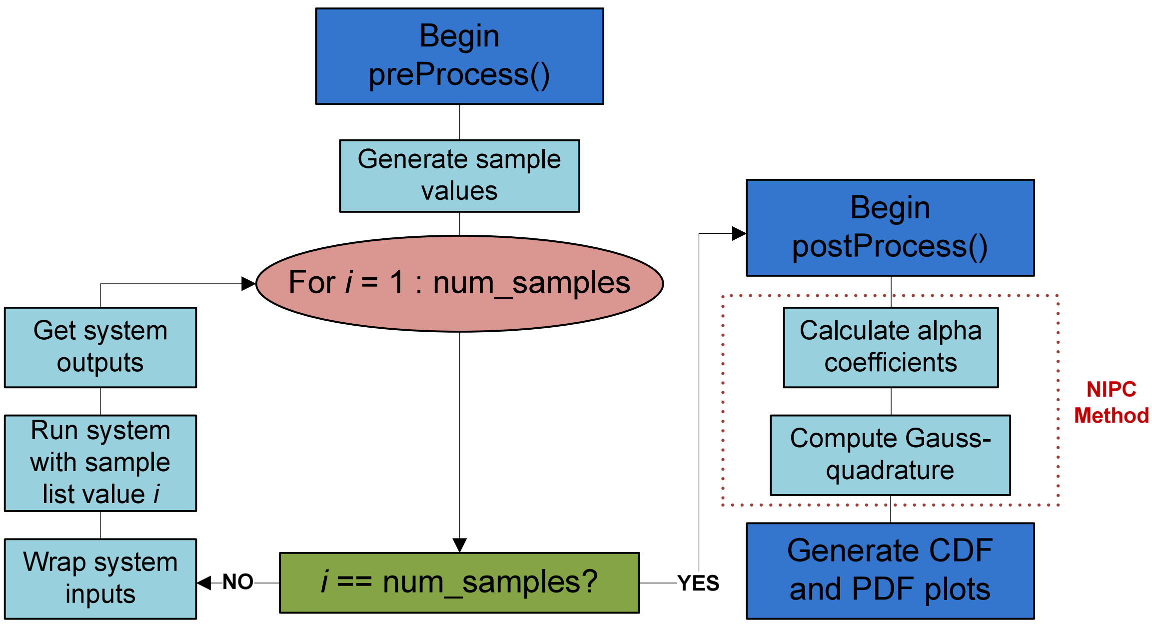


**Figure 3. Framework.py Process Overview (shown for NIPC method)**

### C. Framework Capabilities – NIPC Techniques

The postProcess() section of the UQ Framework applies the NIPC methodologies to the output data generated from the user-specified (black box) system. This procedure can be summarized by the following bulleted list:

postProcess()

- Pure Aleatory Analysis
    - NIPC Methods
        - Regression
            - Latin Hypercube Sampling
        - Quadrature
            - Quadrature Points
    - Calculate Expansion Coefficients
    - Calculate Mean, Standard Deviation, PDF, CDF, etc.
- Mixed Aleatory-Epistemic Analysis (Second-Order Probability)

- NIPC Response Surface Formulation
    - Regression
        - Latin Hypercube Sampling
    - Quadrature
        - Quadrature Points
- Calculate Expansion Coefficients
- Formulate Stochastic Response Surface
- Apply Response Surface to Second-Order Probability

For clarity, Figure 4 was included to illustrate this process.

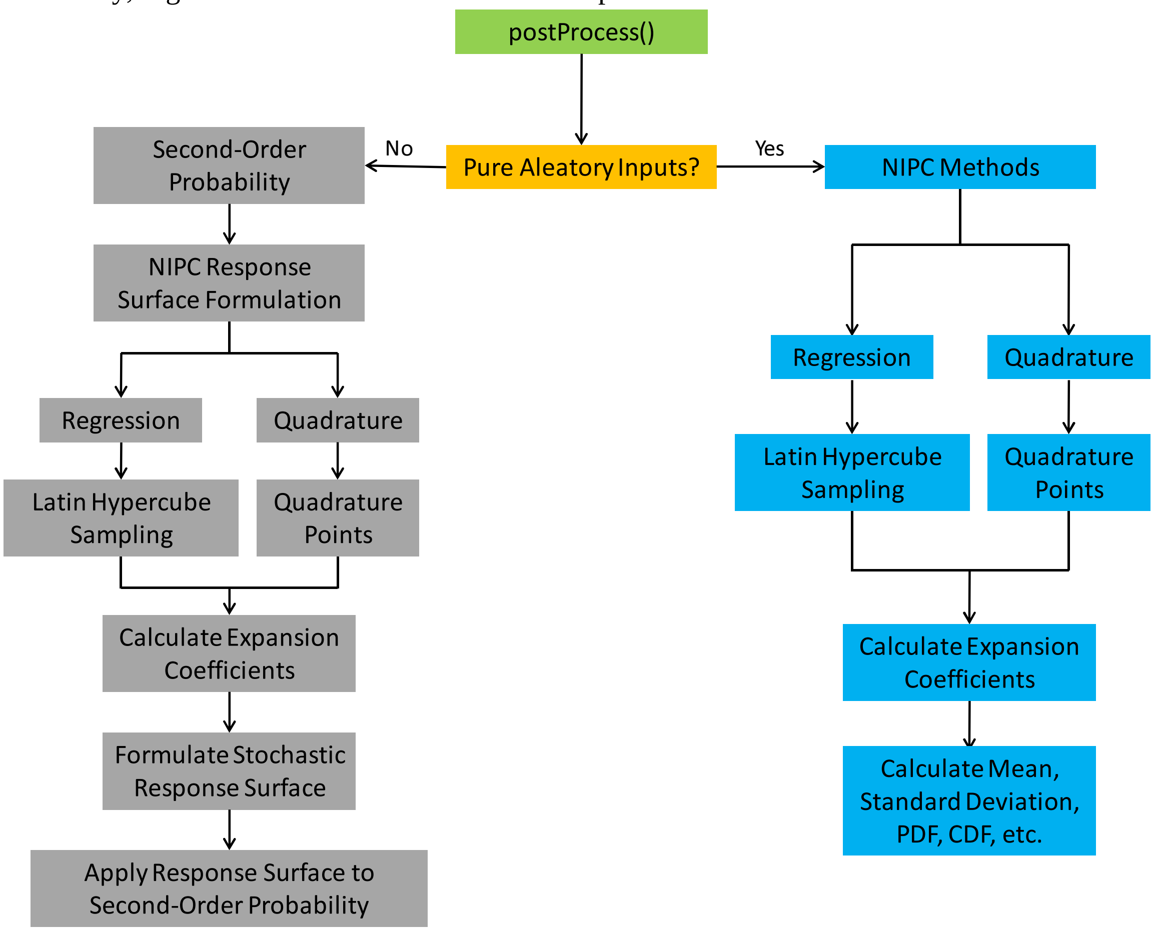


**Figure 4. Flowchart for NIPC Methodologies with UQ Framework**

## VI. UQ of MOOL Reusable Launch Vehicle System

### A. Integrated Spacecraft System – RLV Demonstration Application

The MOOL RLV analysis framework application includes Modules (objects) to handle the disciplines of Geometry, Aerodynamics, Trajectory, Thermal, Structural Optimization, Mission Performance, and Optimization. These modules were implemented using off-the-shelf software in most cases, with some custom code developed as required. The configuration used in this hypersonic process is a vehicle configuration from the Air Force High Alpha RLV Aerodynamic Configuration Development Program. The purpose of the development program was to perform tests and validate the CFD codes used for predicting airflow around the six different vehicle configurations being researched. The RLV configuration used in this MDAO process is shown in Figure 5. This process is discussed in detail below. The implemented RLV process seamlessly handles the passing of data between modules. Ultimately, this allows system level optimization and trade studies to be performed.

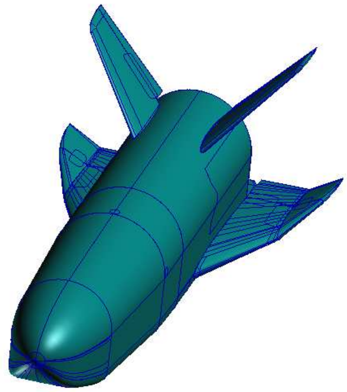

**Figure 5. RLV Vehicle Configuration F.**

The overall process for MDAO of the RLV is shown schematically in Figure 6. The process is shown in Design Structure Matrix format, in which the inputs to a module are shown in the same column as the module, and the outputs are shown on the same row (with the exception of the far left column, which represents overall process inputs). For example the FEM Mesh (on the same row as Geometry and the same column as Structural) is generated by the Geometry module and used by the Structural module. The modules are listed in execution order along the diagonal, starting with geometry and ending with mission performance. Only the most important interactions are shown.

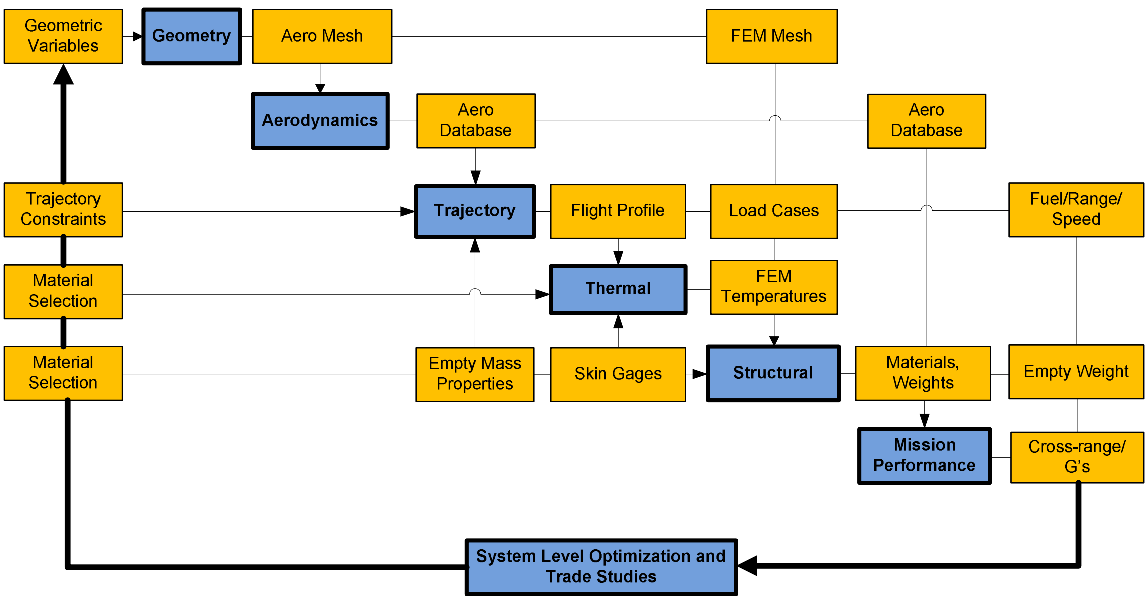


**Figure 6. RLV design structure matrix.**

## B. Modules Included in MOOL RLV System

The following subsystem modules were incorporated into the RLV system model:

1. **Geometry -** The Geometry Module is responsible for morphing analytical meshes for use by the MOOL system.
2. **Aerodynamics -** The Aerodynamics Module is responsible for calculating aerodynamics (in the form of an aerodynamic database) for a reusable launch vehicle configuration.
3. **Trajectory** – The Trajectory Module is responsible for determining the optimum trajectory for three types of launch vehicle trajectories. These include ascent, descent, and boost/flyback.
4. **Thermal** – The Thermal Module is responsible for calculating the temperature response of all structural components subjected to aero heating and to determine the necessary material thicknesses for the thermal protection system (TPS).
5. **Structural -** The Structural Module is responsible for analyzing a user-supplied structural model and optimizing the structural sizing to minimize the structural weight.
6. **Mission Performance -** The Mission Module provides the system-level integration of the results from the steady aerodynamics, propulsion/power, and structural optimization modules to evaluate the overall performance of the vehicle.
7. **Optimization** – The Optimization Module is responsible for building tools for sample point generation by using Design of Experiments (DOE), creating metamodels (surrogate models) for multi-fidelity correction functions, and performing design optimization.

The MOOL RLV system serves to provide system-level performance analysis of a particular RLV design configuration. System-level design and optimization can be carried out fairly effortlessly using this analysis framework.

### C. Description of the Stochastic Problem

The MOOL RLV system has a large number of input parameters. Many of these parameters are propagated through multiple subsystem modules and can have a significant impact on the overall output of the system. Uncertainties (inherent and epistemic) can also be associated with some of the input parameters which can greatly affect the output of the MOOL RLV system analysis. For the demonstration problem of applying the efficient uncertainty quantification techniques to an integrated space system, there were a total of three input uncertainties selected. Two inherent (aleatory) parameters and one epistemic (model form) uncertainty were chosen. The altitude of the initial re-entry point was selected as one inherent uncertainty. The re-entry altitude, which can be thought of as an initial condition, was selected as an uncertainty source because a small deviation from the nominal altitude can greatly affect the overall trajectory of the re-entry vehicle which also gets propagated through many of the subsystem modules. The altitude was assumed to have a normal distribution with a mean of 295,000 feet and a standard deviation of 1,666.67 feet. The second inherent uncertainty selected was the re-entry angle of attack ($\alpha$). The parameter $\alpha$ was assumed to have a normal distribution with a mean of 60° and a standard deviation of 1.667°. The third uncertain parameter was chosen to be the Young's Modulus. The RLV system is a futuristic concept vehicle, and so there is some level of uncertainty in the technological advances in structural materials which will be used to construct these types of vehicles. Therefore, the structural property, Young's Modulus, was treated as an epistemic uncertainty to account for the unknown material that will be used in future manufacturing of the RLV. The lower and upper bounds for the Young's Modulus was selected as 25,000,000 psi and 29,600,000 psi, respectively. An overview of the input uncertainties is shown in Table 1.

The mixed uncertainties were propagated through the MOOL RLV system to a total of four output variables which are of interest in the design of a RLV system. The four output variables analyzed were the maximum range, maximum cross range, maximum dynamic pressure (q), and the takeoff gross weight (TOGW).

**Table 1. Uncertainty ranges for the parameters used in the RLV problem.**

| Uncertain Parameter | Uncertainty Type | Uncertainty Range |
|---|---|---|
| Altitude | Aleatory (normal) | $\mu$ = 295,000 ft, $\sigma$ = 1,666.67 ft |
| $\alpha$ | Aleatory (normal) | $\mu$ = 60°, $\sigma$ = 1.6667° |
| Young's Modulus | Epistemic | [25,000,000 psi, 29,600,000 psi] |

### D. Mixed Uncertainty Quantification for the RLV Application

The approach described previously was followed to propagate the mixed (aleatory and epistemic) uncertainty through the RLV application problem. For this particular case, Point-Collocation NIPC with an oversampling ratio of two was utilized to formulate the response surface which was implemented into the sampling loops of Second-Order Probability. Convergence studies were carried out and it was found that a 4th order polynomial chaos was sufficient for convergence of the NIPC response surface. This required a total of 70 MOOL RLV system evaluations (Equation 2). A Latin Hypercube Sample (LHS) of size 1,000 was used for the outer loop (epistemic) sampling. For each iteration of the outer loop in Second-Order Probability, the NIPC response surface was utilized for the inner loop (aleatory) UQ, with 1,000 samples, which produced a single cumulative distribution function (CDF). The overall Second-Order Probability analysis produced 1,000 CDF curves, which were then evaluated to find the upper and the lower bounds of the output variables of interest at various probability levels.

The interval bounds at various probability levels for each of the four output variables of interest (maximum range, maximum cross range, maximum dynamic pressure, and TOGW) are shown in Table 2. The interval range for maximum range, maximum cross range, and maximum dynamic pressure (q) are much smaller compared to the interval range of TOGW. This result implies that the epistemic uncertainty (Young’s Modulus) has the largest impact on TOGW. However, the maximum range, maximum cross range, and maximum dynamic pressure have a significant amount of uncertainty due to the aleatory (inherent) input uncertainties. There is a relatively large interval range for TOGW at all probability levels. This result directly implies that the Young’s Modulus (epistemic uncertainty) has a significant contribution to the uncertainty in the vehicle’s TOGW. Uncertainty in TOGW is important from a design point of view because a vehicle’s takeoff weight directly affects the vehicle’s capacity for carrying payload, fuel, etc.

**Table 2. Interval bounds for the output variables of interest at various probability levels.**

| Probability Level | Maximum Range (miles) | Maximum Cross Range (miles) | Maximum q (psf) | Maximum TOGW (lbs) |
|---|---|---|---|---|
| P = 0.05 | [985.33, 987.65] | [1158.36, 1160.52] | [126.19, 146.59] | [26304.16, 28421.77] |
| P = 0.2 | [988.17, 988.82] | [1163.20, 1164.10] | [174.83, 177.30] | [27727.63, 29523.33] |
| P = 0.4 | [989.15, 990.17] | [1163.91, 1164.33] | [194.67, 195.97] | [28045.59, 29915.57] |
| P = 0.6 | [989.91, 991.26] | [1164.34, 1164.64] | [206.68, 207.97] | [28270.58, 30287.93] |
| P = 0.8 | [990.92, 992.00] | [1164.57, 1165.47] | [215.50, 217.97] | [28639.11, 30600.30] |
| P = 0.95 | [994.74, 995.35] | [1164.80, 1166.65] | [224.13, 234.68] | [30271.89, 33030.52] |

The results of the mixed (aleatory-epistemic) uncertainty quantification can be used in the assessment of the robustness or the reliability of a given vehicle. For example, in a robust design study where aleatory and epistemic uncertainties are present, one possible approach would be to minimize the variation (interval) at the mean probability level (p=50%). By shrinking this interval, the design sensitivity due to the epistemic uncertainties would be reduced. One method for reducing the interval is by gaining a better fundamental understanding of the physics associated with the epistemic uncertainty, and developing more accurate physical models. Alternatively, the designs that are robust to the uncertainty in physical models can be developed. In a reliability-based assessment, a large interval (high epistemic uncertainty) at a specified probability level may indicate a larger design failure region for a given vehicle configuration and flight condition, which has to be addressed again with a stochastic design framework.

The demonstration problem shown here has successfully displayed the efficiency of the developed uncertainty quantification methods. In the current problem, all relevant results were obtained with 70 evaluations of the MOOL RLV system. If traditional and existing second-order probability methods were used to achieve these same results, it would require a total of $10^6$ MOOL RLV system evaluations. A conservative estimate of the runtime of one MOOL RLV evaluation is approximately five hours. Using this estimate, the current method of second-order probability with the NIPC response surface implementation took approximately 15 days to complete. If the traditional/existing methods were used then it would take approximately 570 years to complete all the simulations. The time requirement for propagating mixed uncertainties using the traditional methods is obviously not feasible. However, the second-order probability with the NIPC response surface formulation makes it feasible to propagate mixed uncertainties in a relatively reasonable amount of time.

## VII. Conclusions and Future Work

The development of an efficient uncertainty quantification framework with application to a complex reusable launch vehicle system has been presented. The UQ capabilities and layout of the developed framework have been discussed. Specifically, the pure-aleatory NIPC and mixed (aleatory-epistemic) second-order probability methods have been detailed and subsequently applied to an RLV system. The probability levels were given for four output variables of interest from the RLV system in which three input uncertainties were propagated. The epistemic uncertainty was shown to have the largest impact on the TOGW. Additionally, the NIPC methods utilized within the developed UQ Framework have proven to be efficient relative to existing/traditional techniques as seen in the RLV demonstration problem.

The UQ Framework being developed is much a work in progress. A key technical objective for the future effort is the refinement of NIPC methods to further improve their computational efficiency and accuracy for mixed uncertainty propagation in spacecraft system models. Additionally, a non-linear global sensitivity analysis capability will be integrated to the uncertainty quantification framework to rank the importance of each uncertainty source and to reduce the number of dimensions in uncertainty space. The development of an adaptive uncertainty quantification methodology for problems with a large number of uncertain variables which will successively utilize different NIPC methods depending on the size of the problem along with the global sensitivity information. Next, a general Quantification of Margins and Uncertainties methodology will be integrated to the uncertainty quantification framework which will include (1) the consideration of both aleatory and epistemic forms in the calculation uncertainty and margins, (2) the utilization of response surfaces based on NIPC for the propagation of uncertainty through each sub-system and overall system, and (3) robust measures to describe the sub-system and overall system safety/reliability/robustness which can be used in decision-making and mission planning. These capabilities will be available for usage in an advanced GUI also to be developed. Lastly, integration of the UQ framework with advanced MDAO software will allow for UQ analysis of an enormous amount of already-developed systems.

## VIII. Acknowledgments

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## IX. References

[1]Hosder, S. and Walters, R., "Non-Intrusive Polynomial Chaos Methods for Stochastic CFD - Theory and Applications," Symposium on computational Uncertainty in Military Vehicle Design NATO Applied Vehicle Technology Panel, Paper No. 47, Athens, Greece, December, 2007.

[2]Eldred, M., Swiler, L., "Efficient Algorithms for Mixed Aleatory-Epistemic Uncertainty Quantification with Application to Radiation-Hardened Electronics," Sandia National Laboratories Report, SAND2009-5805, September, 2009.

[3]Swiler, L., Paez, T., Mayes, R., Eldred, M., "Epistemic Uncertainty in the Calculation of Margins," AIAA-Paper 2009-2249, 50th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Palm Springs, CA, May 4-7, 2009.

[4]Oberkampf, W. L., Helton, J. C., and Sentz, K., "Mathematical Representation of Uncertainty, AIAA-Paper 2001-1645," 3rd Non-Deterministic Approaches Forum, Seattle, WA, April, 2001.

[5]Oberkampf, W. L. and Helton, J. C., "Investigation of Evidence Theory for Engineering Applications, AIAA-Paper 2002-1569," 4th Non-Deterministic Approaches Forum, Denver, CO, April, 2002.

[6]Fay, J. A. and Riddell, F. R., "Theory of stagnation point heat transfer in dissociated air," Journal of Aeronautical Sciences, Vol. 25, No. 25, February, 1958, pp. 73-85.

[7]Bettis, B., Hosder, S., "Quantification of Uncertainty in Aerodynamic Heating of a Reentry Vehicle due to Uncertain Wall and Freestream Conditions," AIAA Paper 2010-4642, 10th AIAA/ASME Joint Thermophysics and Heat Transfer Conference, June 2010, Chicago, IL.

[8]Wiener, N., *The Homogeneous Chaos*, American Journal of Mathematics, Vol. 60, No. 4, 1938, pp. 897–936.

[9]Xiu, D. and Karniadakis, G. E., *Modeling Uncertainty in Flow Simulations via Generalized Polynomial Chaos*, Journal of Computational Physics, Vol. 187, No. 1, May 2003, pp. 137–167.

[10]Walters, R. W. and Huyse, L., *Uncertainty Analysis for Fluid Mechanics with Applications*, T. rep., ICASE 2002-1, NASA/CR-2002-211449, NASA Langley Research Center, Hampton, VA, 2002.

[11]Hosder, S., Walters, R., and Balch, M., *Efficient Sampling for Non-Intrusive Polynomial Chaos Applications with Multiple Uncertain Input Variables*, AIAA-Paper 2007-1939, 9th AIAA Non-Deterministic Approaches Conference, Honolulu, HI, April 2007, CD-ROM.

[12]Ghanem, R. G., Spanos, P. D., Stochastic Finite Elements: A spectral Approach, Springer-Verlag, New York, 1991.

[13]Python Community, "Python Programming Language Official Website," Python Software Foundation 1990-2011. [http://www.python.org].